



PHYS 101 – General Physics I Midterm Exam 2

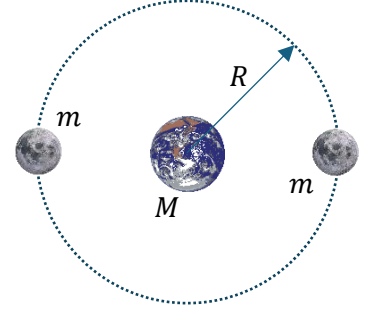
Duration: 90 minutes

Saturday, 12 April 2025; 13:00

1. Two identical moons, both of mass m , are orbiting around a planet of mass M . The moons are in a circular orbit of radius R and at diametrically opposite points of the circle. The mass m of the moons is close to the mass M of the planet, so the gravitational attraction between the moons is not negligible.

(a) (10 Pts.) What is the net gravitational force magnitude on one of the moons?

(b) (15 Pts.) What is the period of rotation of the moons around the planet?



Solution:

(a) Newton's law of universal gravitation:

$$F = G \frac{m_1 m_2}{r_{12}^2}$$

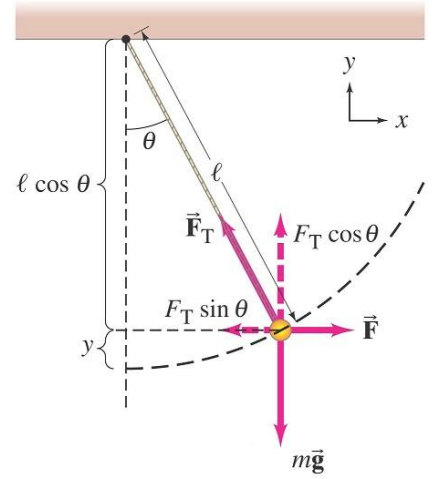
(b) The magnitude of the gravitational force on each moon is

$$F = G \frac{Mm}{R^2} + G \frac{m^2}{4R^2} \rightarrow F = G \frac{m \left(M + \frac{m}{4} \right)}{R^2}$$

Since moons are in circular orbit, by Newton's second law, we have

$$F = m \frac{v^2}{R}, \quad v = \frac{2\pi R}{T} \rightarrow G \frac{m \left(M + \frac{m}{4} \right)}{R^2} = \frac{4\pi^2 m R}{T^2} \rightarrow T = \sqrt{\frac{4\pi^2 R^3}{G \left(M + \frac{m}{4} \right)}} = 2\pi \sqrt{\frac{R^3}{G \left(M + \frac{m}{4} \right)}}$$

2. A simple pendulum consists of a small object of mass m suspended by a cord of length ℓ of negligible mass. A force $\vec{F}$ is applied in the horizontal direction, moving the mass very slowly so the acceleration is essentially zero. (Note that the magnitude of $\vec{F}$ will need to vary with the angle θ that the cord makes with the vertical at any moment, and all three forces acting on the mass, along with the components of the tension force are shown in the figure.)



- (a) (6 Pts.) Find the magnitude of the force $\vec{F}$ as a function of the angle θ .
- (b) (7 Pts.) Determine the work done by this force, $\vec{F}$, to move the pendulum from $\theta = 0$ to $\theta = \theta_0$.
- (c) (7 Pts.) Determine the work done by the gravitational force on the mass.
- (d) (5 Pts.) Determine the work done by the force $\vec{F}_T$ that the cord exerts on the bob.

Solution:

(a)

$$F - F_T \sin \theta = 0, \quad F_T \cos \theta - mg = 0 \quad \rightarrow \quad F = mg \tan \theta$$

(b)

$$W_F = \int_0^{\theta_0} \vec{F} \cdot d\vec{r} = \int_0^{\theta_0} F \cos \theta \ell d\theta = mg\ell \int_0^{\theta_0} \sin \theta d\theta = mg\ell(1 - \cos \theta_0)$$

(c)

$$W_G = \int_0^{\theta_0} m\vec{g} \cdot d\vec{r} = mg\ell \int_0^{\theta_0} \cos\left(\theta + \frac{\pi}{2}\right) d\theta = -mg\ell \int_0^{\theta_0} \sin \theta d\theta = mg\ell(\cos \theta_0 - 1)$$

One can obtain the same result using the expression of the potential energy associated with the gravitational force.

$$\Delta U_G = mg(\ell - \ell \cos \theta_0) = -W_G$$

(d) Since $\vec{F}_T \cdot d\vec{\ell} = 0$, we have $W_{F_T} = 0$.

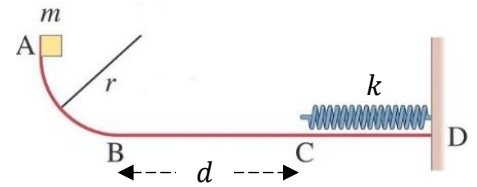
Note that

$$W_{\text{net}} = W_F + W_G + W_{F_T} = 0,$$

and since $W_{F_T} = 0$, we have $W_F = -W_G$.

3. (25 Pts.) Consider the track shown in the figure. The section AB is one quadrant of a circle of radius $r = 5$ m, and is frictionless. B to C is a horizontal span of length $d = 18$ m, with a coefficient of kinetic friction $\mu_k = 0.1$. The section CD under the spring is frictionless. The stiffness constant of the spring is $k = 8$ N/m. A block of mass $m = 1/2$ kg is released from rest at A and slides down the track. Gravitational acceleration is $g = 10$ m/s². Answer the following questions by writing your answers in the boxes provided. Contents of the boxes will be graded.

(a) What is the speed of the block at point B? (b) How much work is done by the friction force as the block moves from B to C? (c) What is the speed of the block at point C? (d) How long does it take the block to move from B to C? (e) What is the maximum compression of the spring?



Solution:

(a)

$$\frac{1}{2}mv_B^2 = mgr \rightarrow v_B = \sqrt{2gr} = 10 \text{ m/s}$$

(a) $v_B = 10 \text{ m/s}$

(b)

$$f_k = \mu_k mg = 0.5 \text{ N}, \quad W_f = -df_k = -9 \text{ J}$$

(b) $W_f = -9 \text{ J}$

(c)

$$\frac{1}{2}mv_C^2 - \frac{1}{2}mv_B^2 = W_f \rightarrow v_C = 8 \text{ m/s}$$

(c) $v_C = 8 \text{ m/s}$

(d)

$$a = -\mu_k g = -1 \text{ m/s}^2, \quad v_C = v_B + at \rightarrow t = 2 \text{ s}$$

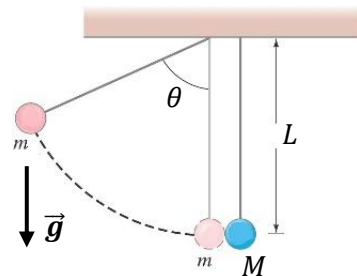
(d) $t = 2 \text{ s}$

(e)

$$\frac{1}{2}mv_C^2 = \frac{1}{2}k\Delta^2 \rightarrow \Delta = \sqrt{\frac{m}{k}} v_C = 2 \text{ m}$$

(e) $\Delta_{\max} = 2 \text{ m}$

4. Two balls, of masses m and M , are suspended as shown in the figure. The lighter ball is pulled away to an angle $\theta = \pi/3$ with the vertical and released. ($\cos(\pi/3) = 1/2$)
 (a) (5 Pts.) What is the velocity of the lighter ball before impact? (b) (10 Pts.) What is the speed of each ball after the elastic collision? (c) (10 Pts.) What will be the maximum height of each ball (measured from the point of collision) after the elastic collision?



Solution:

(a) Total mechanical energy is conserved during the first part of the motion (before the collision).

$$mgL\left(1 - \cos\frac{\pi}{3}\right) = \frac{1}{2}mgL = \frac{1}{2}mv^2 \rightarrow v = \sqrt{gL}$$

(b) During the collision, both horizontal component of the linear momentum and kinetic energy of the system is conserved.

$$mv = mv' + MV', \quad \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}MV'^2 \rightarrow v + v' = V'$$

Solving these two equations for the unknowns v' and V' , we get

$$mv' + MV' = mv, \quad -v' + V' = v \rightarrow V' = \frac{2mv}{m+M} = \frac{2m\sqrt{gL}}{m+M}, \quad v' = \frac{m-M}{m+M}\sqrt{gL}$$

Since $m < M$, we have $v' < 0$, meaning that the ball with mass m bounce back after the collision.

(c) Using the conservation of energy principle for the two masses, we get

$$mgh_m = \frac{1}{2}mv'^2 \rightarrow h_m = \frac{v'^2}{2g} = \left(\frac{m-M}{m+M}\right)^2 \left(\frac{L}{2}\right)$$

$$mgH_M = \frac{1}{2}MV'^2 \rightarrow H_M = \frac{V'^2}{2g} = \frac{2m^2L}{(m+M)^2}$$